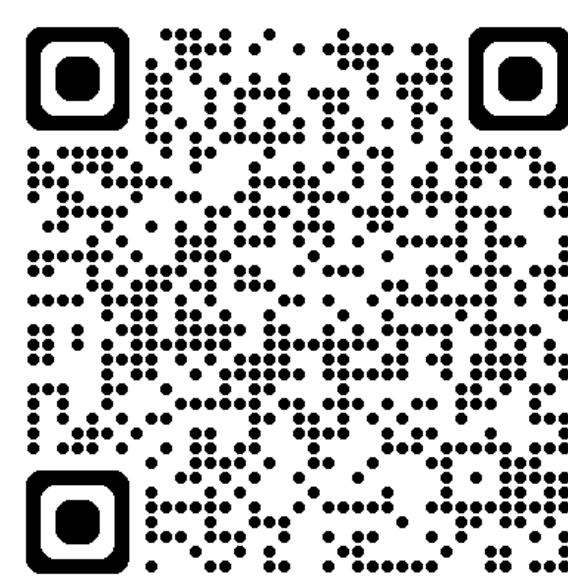


On Cell-Size Distributions in Poisson-Voronoi Tessellations and Resource-Sharing Fractions in Cellular Networks



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The Problem

Zero-Cell Bias

On average, zero cells are larger than typical cells.
Therefore, a randomly selected user is more likely to be located in a larger cell.

Why Does It Matter?

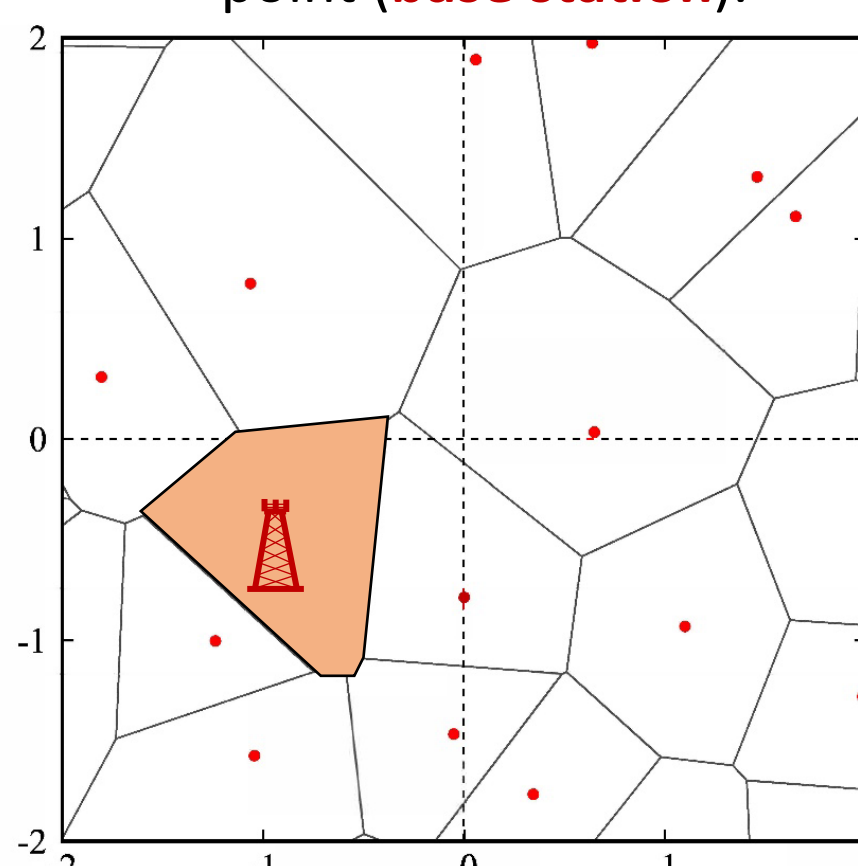
Larger cells tend to contain more active users.
These users compete for the same resources, reducing the transmission opportunity.

Analytical Challenge

The zero-cell area distribution is needed for user-centric performance analysis.

Typical Cell

the Voronoi cell associated with a randomly selected generating point (**base station**).



Zero Cell

the Voronoi cell containing a randomly selected location (**user**) on the plane.

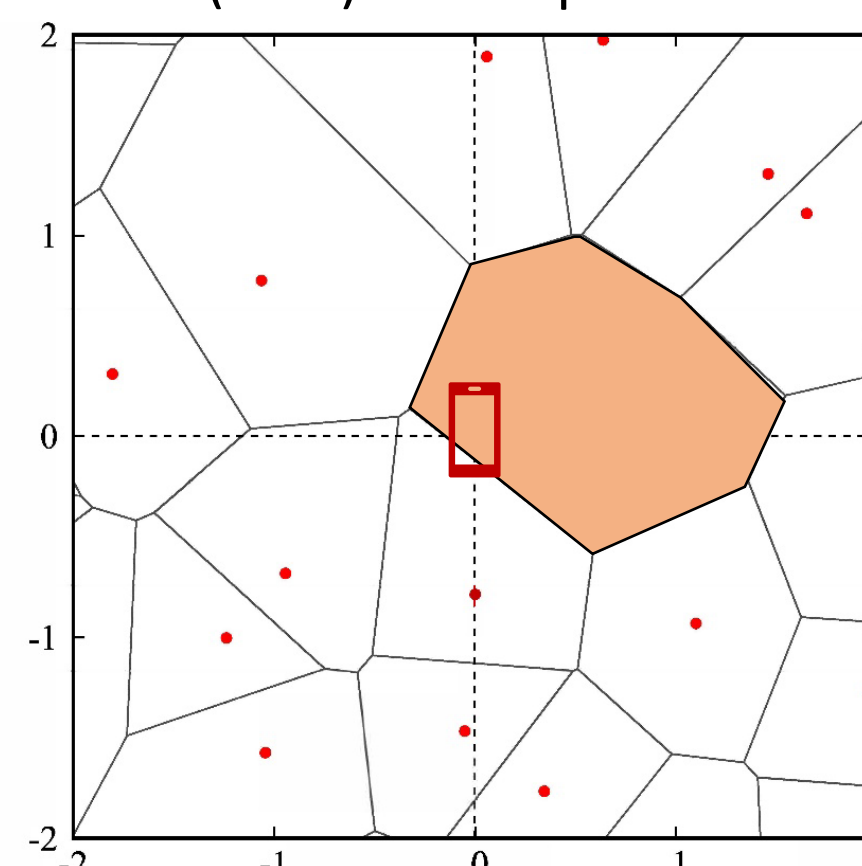


Fig. 1: Typical and zero cells.

Current Solutions

Gamma-Family Approximations for the Typical-Cell Area Distribution

- Ferenc-Néda

$$f(x) = \frac{(\lambda\alpha)^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda\alpha x}, \alpha = 3.5$$

- Tanemura

$$f(x) = \frac{a(b\lambda^a)^{c/a}}{\Gamma(c/a)} x^{c-1} e^{-(b\lambda^a)x^a}$$

$$a=1.080, b=3.032, c=3.311$$

Lack of a Tractable Approximation for the Zero-Cell Area Distribution

Our Contribution

Transformation

Applying Palm theory to derive a zero-cell area approximation from a typical-cell approximation.

Palm Transform

$$f_0(x) = \lambda x f(x)$$

f_0 : zero-cell PDF
 f : typical-cell PDF
 λ : BS density

Transform of Ferenc-Néda Approximation

$$f_0(x) = \frac{\lambda(\lambda\alpha)^\alpha}{\Gamma(\alpha)} x^\alpha e^{-\lambda\alpha x}, \alpha = 3.5$$

Refinement

Refining the model parameters by moment matching or MLE to improve the approximation accuracy.

Before Refinement

$$\alpha = 3.5 \quad (\text{original parameter value in Ferenc-Néda})$$

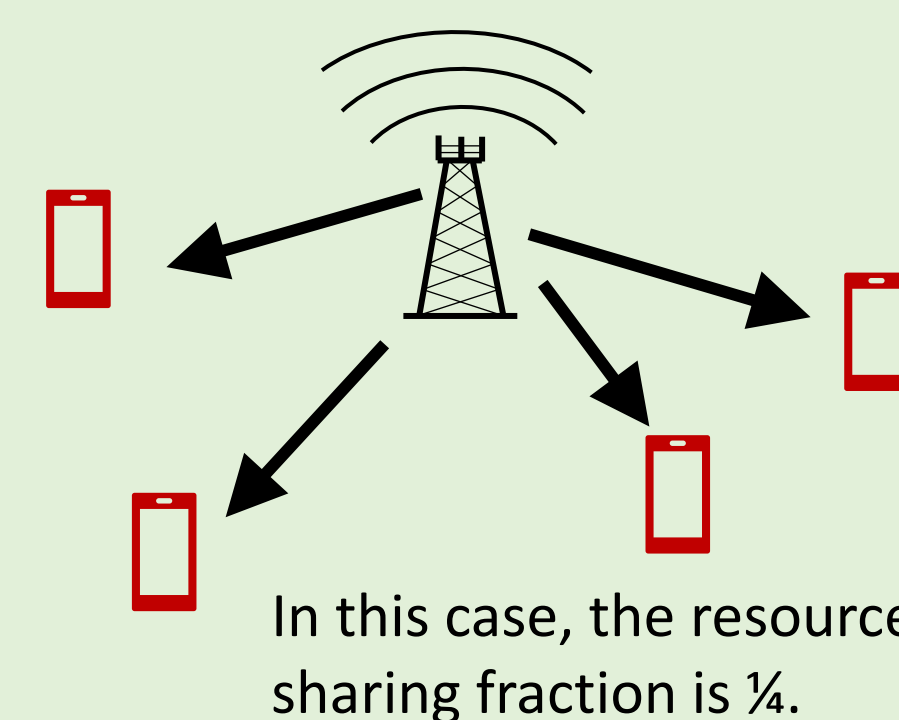
After Refinement

$$\alpha = 3.5692 \quad (\text{moment matching})$$

$$\alpha = 3.5545 \quad (\text{MLE})$$

Evaluation

Evaluating the resource-sharing fraction to **quantify the impact of zero-cell bias**.



Result 1

Relative errors versus simulation are below 0.1%.

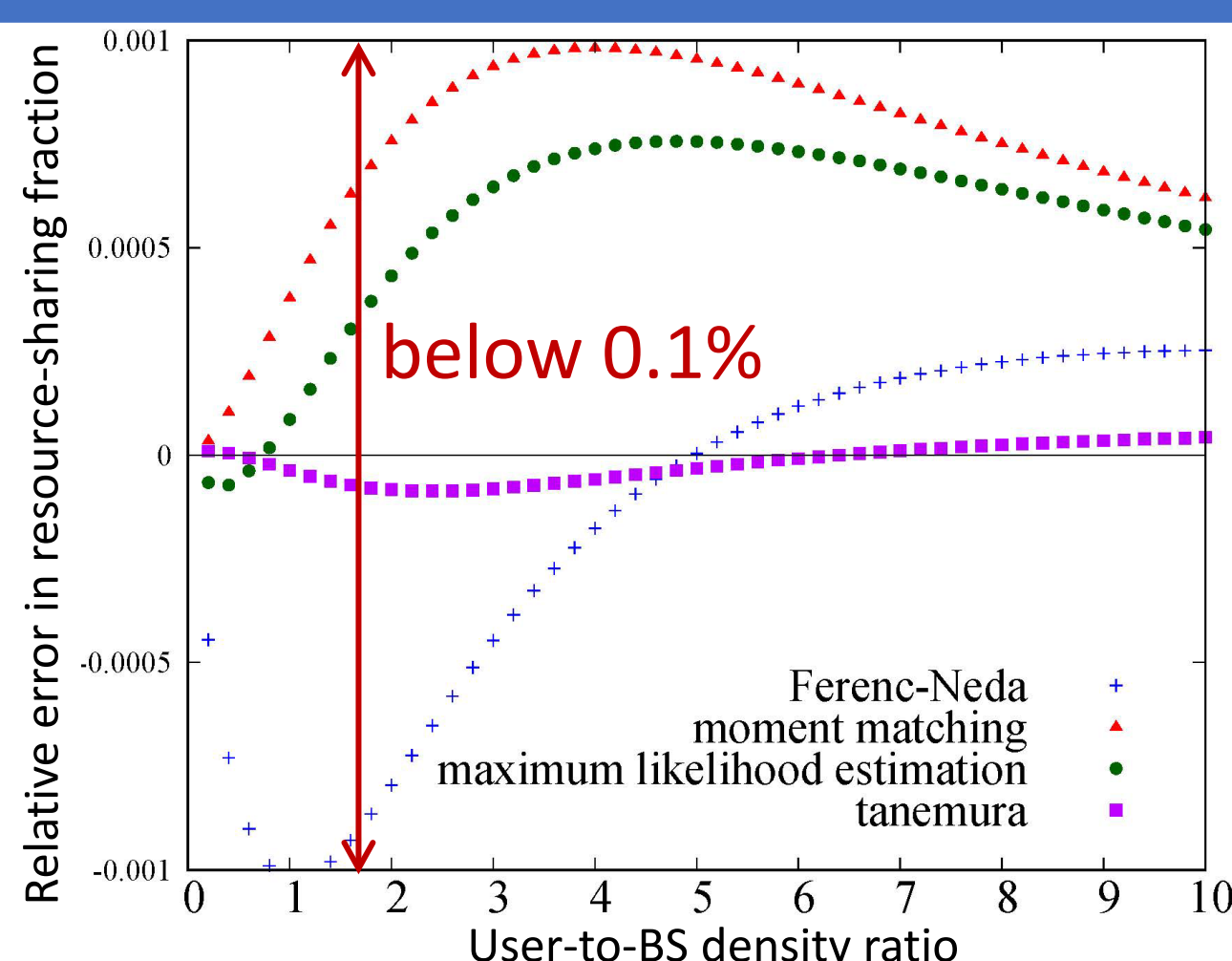


Fig. 2: Relative errors in the resource-sharing fraction.

Result 2

Zero-cell bias cannot be ignored in resource-sharing analysis.

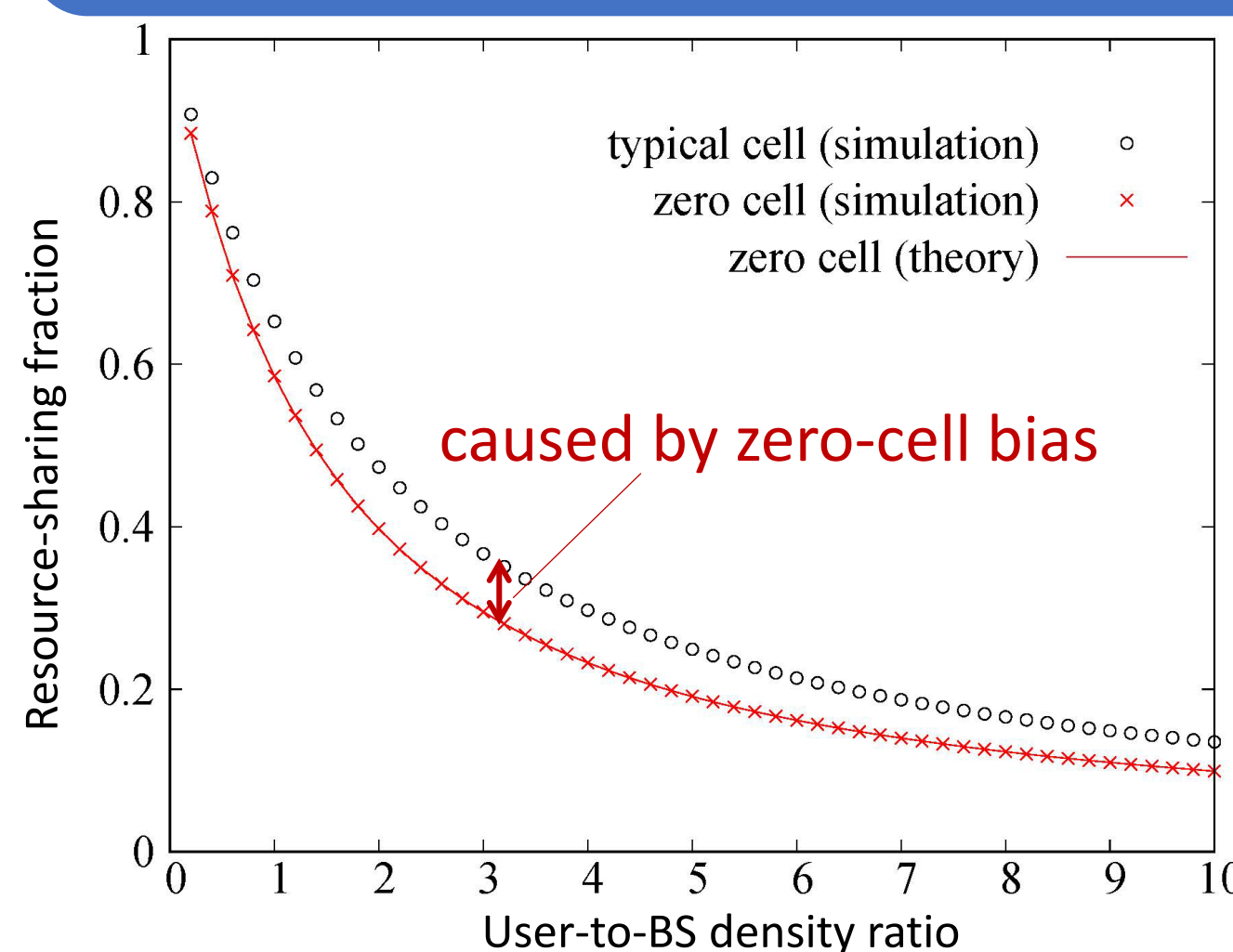


Fig. 3: Resource-sharing fractions based on the typical and zero-cell area distributions.

Next Steps

Zero-cell area distribution conditioned on serving distance:

Quantifying how user-to-BS distance affects user-perceived performance.

Performance analysis of mMTC:

Jointly accounting for zero-cell bias and interference from simultaneously transmitting devices.